Pi Calculation Robot Worksheet

http://www.edhs.org/other/pi/

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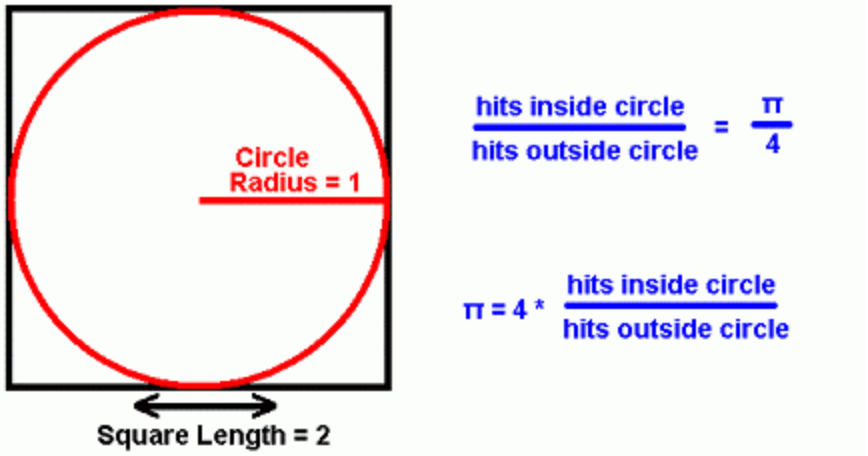
Background:

The Monte Carlo Method of Pi calculation is one of the easiest to understand and code. The price paid for this simplicity is that the Monte Carlo Method takes many many trials to gain substantial decimal accuracy of Pi calculation. Being a "brute force" method of calculation, it takes hundreds of billions of trials just to get five decimal places of Pi accuracy!

The Monte Carlo Method of Pi calculation is based on a simple mathematical relationship. Imagine that a circle with a radius of **1 unit** is inscribed within a square with each side having a length of **2 units** (see diagram below). The area of the circle would be calculated: **circle area** = πr^2. Since the circle radius is 1, the area of the circle is then: **circle area** = π. The area of the square is calculated as: **square area = 4**. So the ratio of the area of the circle to the area of the square is $\pi/4$. This relationship: $\pi/4$ also happens to be equal to the ratio of random hits inside the circle to random hits outside the circle. With some reorganization, we arrive at the formula: π **= 4 * hits inside circle / hits outside circle.**



Assignment:

(1) Run the following trial sample sizes making sure to keep the file **PiCalcResults.txt** for future reference:

a) 100 trials (estimated time < 1 second)
b) 10,000 trials (estimated time < 1 second)
c) 1,000,000 trials (estimated time < 8 second)
d) 100,000,000 trials (estimated time ± 80 second)
e) 10,000,000,000 trials (estimated time ± 3 hours)

(2) Visit the website and report the **two** runs you completed that were at least 100 million trials in size. If possible, please run your computer overnight and attempt between 10 billion and 100 billion trials. The larger the runs, the more the "pooled data" will benefit from your efforts. Please report your results.

(2) Create a graph* showing **Digits Of Pi Accuracy** on the X axis against **Log_{10} Of Total Trials** on the Y axis. Use the data from the trial sample sizes listed above. For reference purposes, Pi to 30 digits of decimal accuracy is: **3. 14159 26535 89793 23846 26433 83279**

*** = For printable graph paper, visit: http://www.mathematicshelpcentral.com/graph_paper.htm**
Use semi-logarithmic paper for this graph. Ask you teacher how semi-logarithmic paper is used or visit http://www.physics.uoguelph.ca/tutorials/GLP/ for a quick tutorial.

Discussion Questions:

Using your data, and summary data provided on the "view" page at `http://www.edhs.org/other/pi/`, answer the following questions.

(1) Based on your research, the trials you have run, and the graphs you have created, what are the major limitations of the Monte Carlo Method of Pi Calculation?

(2) Even though the Monte Carlo Method has limitations, approximately how many trials would be needed to reach a useful "every day" accuracy of Pi (4 digits of accuracy) based on your data?

(3) As of today, please visit the "VIEW" web page for this project on the internet (see address above) and provide the following current information:

A) Hours Of Computer Time:

B) Total Trials:

C) Pi Estimate So Far:

(4) Based on the results presented on the VIEW page of the website, on average, how many Trials Per Second have been performed? Use the formula:

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Trials Per Second = Total Trials / (Hours Of Computer Time * 3600)
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(5) Based on your graph of **Digits Of Pi Accuracy** against **Number Of Trials Run,** and summary data from the website, extrapolate a reasonable estimate as to how many trials would be required to gain **8 digits** of Pi accuracy.

(6) Assuming that you could run about 1.8 million trials per second, how much time would be required to gain **8 digits** of Pi accuracy. Use the number of trials you estimated in the previous question as the basis for this answer. Report time in years (hint) using the formula: `Years = Seconds / 31,556,736`.